Dynamic Modelling and Analysis of Sedimentation-Consolidation Model in a Paste Thickener

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ABSTRACT
The introduction of paste thickener to the mining industry has attracted considerable attentions due to its potential economic and environmental benefits. Paste thickeners inherit the ability to deliver higher product concentration than that of conventional thickeners, which implies potential improvements in water savings. In this paper, a mathematical dynamic model for process control purpose is developed and dynamical features of paste thickeners are studied. The model is discretised and linearized to facilitate analysis. Values of parameters, which in essence related to material properties, are explained and determined. Analysis shows that the system behaves differently at different discretised cells. In addition, two distinct dynamics, the fast and slow dynamics, are observed in the system which corresponds to the hindered settling and compression zones respectively. Simulation results show that the model output is in a good agreement with plant data. This shows that the values of parameters used are able to describe the material properties satisfactorily and the validated model can be used to design a control system for future studies.

Keywords: Modelling and Simulation, Control, Mineral Processing, Solid-Liquid Separation.

1. INTRODUCTION
Thickening is the principal means used for solid-liquid separation in mineral processing, coal preparation and water/wastewater treatment plant. Here, thickening of coal tailings will be evaluated within Coal Handling and Preparation Plants (CHPPs). The purpose of thickening is to increase the solids content by removing water and for the purpose of recycling. This is accomplished by utilizing the natural force of gravity to achieve sedimentation, producing a clarified liquid which can be decanted, and leaving a thickened suspension, or underflow product with higher concentration.

Over the past decades, conventional thickeners have been widely used in industry for thickening purpose. However, the thickening capacity of conventional thickeners has reached a bottleneck, in which the maximum solids concentration of 30 wt% [1] delivered can no longer be satisfactory due to the concerns of increasing scarce water resources. Therefore, paste thickeners were introduced by National Coal Board British [2] which possess the ability to deliver higher solids content (> 50 wt%) with smaller equipment size.

Paste thickeners rely on a deeper sediment layer of compressible bed to augment the thickening capacity. As a result, a higher complexity is inherited in the process as the motion of each particle in the compression region is influenced by the presence of the others due to close proximity between particles. Despite the complexity of the process, paste thickeners have still attracted attentions in the industry as it is believed that thickening by providing high volume of bed depth is more economical than those require greater area with a rather shallow bed level as in the case for conventional thickeners [3]. Consequently, theory of sedimentation which assumes ideal suspension is no longer valid to be applied for paste thickeners. Furthermore, the presence of flocculated suspension has driven the development of theory of sedimentation-consolidation recently.

This paper focuses on the dynamic analysis of the system when an interruption is introduced to the paste thickener. The justification of values of parameters involved in the model is also presented, which is important in simulation studies and for control system design. Finally, an open-loop simulation is carried out to ascertain the validity of the mathematical model by comparing to real plant data.

This paper is organized as follows. First, the modelling of sedimentation-consolidation theory is outlined. Then, discretization is performed to convert the model into ordinary differential equations. The responses of the system to changes in inlet stream, outlet stream and coal type are presented together with explanation. Simulation results are compared to plant data followed by a discussion. Finally, a summary and recommendations for future studies are explained in the conclusions.

2. MODELLING
In this paper, two distinct zones are considered in the modelling of a paste thickener: (1) the hindered settling region and (2) the compression zone, which is shown in a
continuous operating paste thickener in Fig. 1. The inlet stream is denoted as \( Q_F \) while the discharge stream is represented by \( Q_D \). The difference between the inlet and outlet flow is the overflow stream, indicated as \( Q_F - Q_D \). The overflow stream is assumed to consist of water with negligible amount of solids present.

The modeling of ideal suspension was first proposed by Kynch [4] which is a major breakthrough in the theory of sedimentation:

\[
\frac{\partial \phi(x,t)}{\partial t} + \frac{\partial }{\partial x} (f_{bk}(\phi)) = 0 \quad \text{for} \quad 0 < x < L \quad \text{and} \quad t > 0, \tag{1}
\]

where \( \phi \), the solids concentration, in volume fraction. The sedimentation flux density function \( f_{bk} \), is assumed to be a function of local concentration only. The theory was developed based on the assumption of uniform particle distribution and is only suitable to be applied in ideal conventional thickeners.

There are several empirical formulae commonly used to express \( f_{bk} \), one of it is by Michaels and Bolger [5], which is a modification of the formula proposed by Richardson and Zaki. [6]

\[
f_{bk}(\phi) = v_\infty \phi \left( 1 - \frac{\phi}{\phi_{\text{max}}} \right)^N, \tag{2}
\]

where \( v_\infty \) is the terminal velocity of flocculated particles in the suspension. \( N \) is referred as the exponent for the function. \( \phi_{\text{max}} \) denotes the maximum attainable solids concentration, which is dependent on the system. In this study, \( \phi_{\text{max}} \) is taken to be 0.66, which agrees with Bürger and Concha [7].

Owing to the assumption of ideal suspension, (1) does not valid for sedimentation-consolidation process due to the presence of pore pressure and interaction between particles in compression zone. Therefore, the model for continuous thickening by taking the interactive effect between particles was developed by Bürger et. al., which is a second order partial differential equation [8]:

\[
\frac{\partial (S(x)\phi)}{\partial t} + \frac{\partial }{\partial x} (Q_D(t)\phi + S(x)f_{bk}(\phi)) = \frac{\partial }{\partial x} \left( S(x) \frac{\partial A(\phi)}{\partial x} \right), \tag{3}
\]

where \( S(x) \) represents the cross sectional area of the system and \( Q_D(t) \) is the underflow volumetric flow rate at \( x = 0 \). \( A(\phi) \) is the consolidation function which can be represented as:

\[
A(\phi) = \int_0^\phi a(s) \, ds , \quad a(\phi) = -\frac{f_{bk}(\phi)\sigma_e'(\phi)}{\Delta \rho g \phi}, \tag{4}
\]

where \( \Delta \rho \) is the density difference between solid and liquid and \( g \) is the acceleration of gravity. The consolidation function can be interpreted as the multiplicative effect between sedimentation \( f_{bk}(\phi) \) and solid stress function \( \sigma_e'(\phi) \). \( \sigma_e(\phi) \) is the effective solid stress function, where one of the general forms is proposed by Tiller and Leu [9]:

\[
\sigma_e(\phi) = \begin{cases} 0 & \text{for} \quad \phi \leq \phi_c, \\ \sigma_0 \left[ \left( \frac{\phi}{\phi_c} \right)^k - 1 \right] & \text{for} \quad \phi > \phi_c, \end{cases} \tag{5}
\]

where \( k \) and \( \sigma_0 \) are parameters to be determined. \( \phi_c \) is the critical concentration at which particles start to interact with each other, forming compression zone. The derivative of (5) is:

\[
\sigma_e'(\phi) = \begin{cases} 0 & \text{for} \quad \phi \leq \phi_c, \\ \frac{\sigma_0 k}{\phi_c} \phi^{k-1} & \text{for} \quad \phi > \phi_c, \end{cases} \tag{6}
\]

From (6), it is observed that that the solid stress function has no effect to the system when the concentration is less than the critical concentration \( \phi_c \). This is the hindered settling region as shown in Figure 1 where the solids exhibit free settling without interaction. In this region, (3) will be reduced to a first order hyperbolic function. In the compression zone, all terms remain and the solution will be a second order parabolic function.

Observation of (3) reveals that 3 boundary conditions are required to solve for the partial differential equation. The boundary conditions are given as:

1. \( \phi(x, 0) = \phi_0(x), \quad 0 \leq x \leq L, \quad t = 0, \tag{7} \)
2. \( (Q_D(t)\phi + S(x)f_{bk}(\phi) - \frac{\partial A(\phi)}{\partial x}) = Q_F(t)\phi_F(t), \quad 0 \leq t \leq T, \quad x = L, \tag{8} \)
3. \( f_{bk}(\phi) - \frac{\partial A(\phi)}{\partial x} = 0, \quad 0 \leq t \leq T, \quad x = 0. \tag{9} \)

Eq. (7) is the boundary condition in time domain. At time \( t = 0 \), the solids concentration is the initial concentration.
This enables the model to compute the concentration profile for \( t > 0 \). Eq. (8) and (9) are boundary conditions in spatial domain. At the feed well, which is at \( x = L \), the paste thickener is fed with fresh inlet with flow rate \( Q_F \leq 0 \) (Note the negative sign indicates that the direction of feed flow is in the opposite direction as defined for the spatial domain), with solids concentration \( \Phi_F \). At the discharge level, which is at \( x = 0 \), (3) reduces to only its convective term. This implies that the bulk flow convective is the mechanism responsible to carry the suspension out of the vessel.

3. METHODOLOGY

3.1 Discretization

The mathematical model used to describe the process as in (3) is a partial differential equation which is complicated to solve. It is also not suitable to be used to develop a control system for future work. Hence, the technique of finite difference discretization is employed to reduce the equation to a number of vertical sections (cells) with a height of \( \Delta x \) = \( L/N \) where \( N \) is the number of cells. Let \( \Phi_j^h \) denotes the approximate solution value of \( \Phi \) at time \( t \) and cell \( j \), i.e. \( \Phi(x_j, t_n) \). The discretized system is given as follows:

\[
\begin{align*}
S_j \frac{\Phi_j^{h+1} - \Phi_j^h}{\Delta t} + Q_D(t_n) \frac{\Phi_j^{h+1} - \Phi_j^h}{\Delta x} + \\
S_j \frac{f_{bk}^E(\Phi_j^{h+1}, \Phi_j^{h+1}, \Phi_j^{h+1}) - f_{bk}^E(\Phi_j^{h-1}, \Phi_j^{h-1})}{\Delta x} + \\
\frac{S_j}{h} [A(\Phi_j^{h+1}) - A(\Phi_j^{h-1})] - \frac{S_j}{h} [A(\Phi_j^{h}) - A(\Phi_j^{h-1})]
\end{align*}
\]

where \( f_{bk}^E(\Phi_i, \Phi_{i+1}) \) is the Engquist-Osher scheme of \( f_{bk} \) [10]:

\[
f_{bk}^E(\Phi_i, \Phi_{i+1}) = f_{bk}(0) + \int_0^{\Phi_i} \max\{f_{bk}(s), 0\} \, ds + \int_{\Phi_i}^{\Phi_{i+1}} \min\{f_{bk}(s), 0\} \, ds
\]

The interest is the concentration at time \( t = n + 1 \), which is \( \Phi_j^{h+1} \). Rearranging the discretized system gives

\[
\begin{align*}
\Phi_j^{h+1} = & \quad \Phi_j^h - \frac{\Delta t}{\Delta x} [Q_D(t_n)(\Phi_j^{h+1} - \Phi_j^h) + S_j f_{bk}^E(\Phi_j^{h+1}, \Phi_j^{h+1}) - \\
& S_j f_{bk}^E(\Phi_j^{h-1}, \Phi_j^{h-1}) - \frac{\Delta t}{(\Delta x)^2} [S_j (A(\Phi_j^{h+1}) - A(\Phi_j^{h})] - \\
& S_j [A(\Phi_j^{h+1}) - A(\Phi_j^{h-1})]
\end{align*}
\]

Eq. (10) is the general equation for the discretized model. To ensure the convergence of the resulting scheme, the following stability criterion must be satisfied:

\[
\frac{1}{\min_{\Phi} \max_{(\Delta x)^2} [Q_D(t)| + S_{max} \max_{\Phi} (|f_{bk}(\Phi)|} + \\
2 \max_{\Phi_{max}} a(\Phi) \frac{\Delta t}{\Delta x} \leq 1
\]

One means to ensure the condition in (11) is satisfied by choosing very small \( \Delta t \). When \( \Delta t \to 0 \), (10) can be expressed as an ordinary differential equation (ODE):

\[
\frac{d\Phi_j}{dt} = \frac{1}{S_j} \left[ \frac{1}{\Delta x} \left[ Q_D(t)(\Phi_{j+1}(t) - \Phi_j(t)) + \\
S_j f_{bk}^E(\Phi_j(t), \Phi_{j+1}(t)) - S_j f_{bk}^E(\Phi_{j-1}(t), \Phi_j(t)) \right] - \\
\frac{1}{\Delta x^2} \left[ S_j [A(\Phi_{j+1}(t)) - A(\Phi_j(t))] - S_j [A(\Phi_j(t)) - A(\Phi_{j-1}(t))] \right] \right]
\]

Incorporating the boundary conditions given in (8) and (9), the process descriptions in the feed (\( j = 0 \)) and discharge (\( j = n \)) are given as follows:

\[
\frac{d\Phi_j}{dt} = -\frac{1}{S_j} \left[ \frac{1}{\Delta x} \left[ Q_D(t)\Phi_j(t) - \\
Q_D(t)(\Phi_{j+1}(t) - \Phi_j(t)) - S_j f_{bk}^E(\Phi_{j-1}(t), \Phi_j(t)) \right] - \\
\frac{1}{\Delta x^2} \left[ -S_j [A(\Phi_j(t)) - A(\Phi_{j-1}(t))] \right] \right]
\]

Eqs. (12), (13) and (14) are the three ODEs that describe the process in the paste thickener, at the feed level and at the discharge point respectively. Upon knowing the initial condition of the process, the ODEs can be solved to obtain the concentration profile of the system. For example, if the paste thickener is started from empty, the initial condition of the system \( \Phi(x, 0) \) is readily known.

Eqs. (12), (13) and (14) can also be linearized at a desired operating point to facilitate the analysis of the model. This will result in a system which is represented in a linear time-invariant (LTI) state space model:

\[
\begin{align*}
\dot{x}' &= Ax' + Bu' \\
y' &= Cx' + Du'
\end{align*}
\]

where \( x' \) represents the states of the system, \( u' \) and \( y' \) denote the input and output of the system respectively. The notation \( x' \) is conventionally used to denote deviation variables. \( A, B, C \) and \( D \) are constant matrices.

4. RESULTS AND DISCUSSIONS

Rearranging (3) gives:

\[
\begin{align*}
\frac{\partial \Phi(x,t)}{\partial t} = & \frac{1}{S(x)} \frac{\partial}{\partial x} (Q_D(t)\Phi) - \frac{1}{S(x)} \frac{\partial}{\partial x} (S(x)f_{bk}(\Phi)) + \\
& \frac{1}{S(x)} \frac{\partial}{\partial x} (S(x)A(\Phi))
\end{align*}
\]

Eq. (15) shows that the dynamic of the system can be modeled by three mechanisms:
i. Convection. This is the transport of bulk flow of the particulate system in the thickeners. Convection is only affected by the flow rate of streams, and is independent of any parameters in (2) and (6).

ii. Sedimentation. This is the settling of particles by the force of gravity. It is dependent on two parameters $v_{co}$ and $N$ and is independent of the flow rate of streams. $v_{co}$ has strong relationship with the density and particle size and $N$ is found to be a function of particle shape.

iii. Consolidation. This mechanism only occurs in the compression zone in which the particles interact with each other, exerting a net pressure on each other and at the same time enhancing the dewatering ability. This mechanism is also independent on the flow rate but on all parameters discussed above.

It is interesting to study the dynamic of the system to have a better understanding of the process. The system is linearized about a desired operating point which greatly facilitates the analysis.

4.1. Dynamic Analysis

This section presents the dynamic behavior of the linearized system to an interruption in steady state condition. A small step change is introduced to the linearized model and three scenarios will be considered: (1) a change in inlet feed solids (2) a change in underflow withdrawal rate (3) a change in parameter (coal type). In this study, the system operates at a bed level of 7 m.

4.1.1 Change in Inlet Feed Conditions

A small step change of inlet feed conditions is introduced to the system. The residence time at different levels of the system, defined as the time taken to reach 99% of the ultimate value, are obtained, as shown in Figure 2.

![Figure 2: Residence time at different height in response to a change in inlet feed conditions](image)

The effect to the system is observed to propagate from top to the bottom of the system (note that $x = 0$ is at the bottom of the paste thickener). At the hindered settling region, the residence time is relatively faster due to the convection mechanism. It can be seen that there is an abrupt increase in the residence time at the top of the bed ($7 \text{ m}$). This is believed to be caused by the formation of network interaction between particles that slowed down the process significantly. At this point, the concentration of particles has just over the critical concentration and particles start to exert a force on each other.

4.1.2 Change in Underflow Withdrawal Rate

The removal of product from the bottom of the paste thickener will cause the system to experience a loss of quantity simultaneously at all cells. It is interesting to look at the dynamic of the system from a different perspective: the fast and slow dynamics, obtained from the poles of the transfer functions. A fast pole corresponds to a fast dynamic, with smaller time constant and vice versa, as shown in Figure 3.

![Figure 3: Fast and slow dynamic analysis (x = slow dynamic, + = fast dynamic)](image)

In the top levels, i.e. from 8-11 m, the system behaves quickly with the same rapidity, which is believed to be caused by the fast convection mechanism. However, once the network interaction is formed in the compression zone, the slow dynamics appears to slow down the system considerably. The slow dynamic will be the dominant effect of the compression zone, causing the bed layer to response much slower. This finding also agrees with Figure 2, where the commencement of compression zone shows a jump in residence time as compared to the hindered settling region. In comparison of residence time, the system responds faster towards a change in underflow removal rate as compared to the previous scenario.

4.1.3 Change in Parameter (Coal Type)

Coal handling plants often treat a range of different material types, depending on the coal seam. This makes the coal tailings material properties to be varying. Thus, the change in parameter (tailings material properties) is closely related to the coal type treated within the coal handling plant. It should be emphasized that the change in coal type is often unavoidable due to the processing of different coal seams or from both, open-cut and underground mine coal. Thus, it is necessary to understand the effect of coal type to the dynamic of the system. The parameter $k$ is chosen to vary
due to its significant effect to the system since it is an exponent in the solid stress function, as shown in (5).

It is examined that the change in parameter \( k \) does not result in any effect in the hindered settling region, as shown in Figure 4. This is due to the absent of effective solid stress in the hindered settling region, as in (6). In contrast, the residence time is found to be the largest at the top of the bed and the response is quicker at the bottom of the paste thickener. This can be explained by considering the weight of solids at the top helps compressing particles at the lower levels, which results in a faster response.

![Figure 4: Residence time at different height in response to a change in parameter](image)

There are several conclusions that can be drawn from this investigation:

i. Different levels of the paste thickener response with different rapidity. This leads to a complex dynamic process in the system.

ii. The fast and slow dynamic of the system conveys important message to the dynamic of the process. This shows the importance of bed level consideration in treating the sedimentation-consolidation model as well as control system development.

iii. The system responses quicker towards a change in underflow withdrawal rate than a change in inlet feed conditions. This finding is particularly important when designing a control system. For example, the underflow discharge rate can be taken as the manipulated variable to control the desired control variable.

iv. The boundary between hindered settling and compression zone has the slowest response, which is most probably due to the commencement of formation of solids particle network at the critical concentration. This particular phenomenon indicates the importance and significance of the bed formation in the sedimentation-consolidation model.

### 4.2. Determination of Parameters

From (2) and (6), there are 5 parameters which are unknown. \( v_\text{so}, N \) are involved in the sedimentation function while \( k, \sigma_0 \) and \( \Phi_c \) exist in the effective solid stress function. It is necessary to obtain their values before further study can be carried out. These parameters affect the dynamic of the system in different ways. The decision of the values of parameters used for simulation studies will be summarized in Table 1.

These values are treated as constants in this study. However, it should be noted that in passing downwards through a thickener, the flocculated particles are subjected to an ever increasing solids stress due to the increasing weight of the compressing particles above. Hence, it is believed that the bed layer will be harder to compress at the bottom, implying a varying parameter. Physically, constant parameters imply that an average value can be used to describe the system such that it is treated as a lumped parameter system.

#### Table 1: Coal parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>5</td>
<td>Primarily dependent on the shape of particles. This parameter can be determined experimentally</td>
<td>[11]</td>
</tr>
<tr>
<td>( v_\text{so} )</td>
<td>(-1 \times 10^{-3} \text{ m s}^{-1})</td>
<td>Terminal velocity of a single particle at infinite dilution</td>
<td>[12]</td>
</tr>
<tr>
<td>( \Phi_c )</td>
<td>0.15 ( \text{v/v} )</td>
<td>Critical concentration at which particles start to interact with each other due to close proximity</td>
<td>[13]</td>
</tr>
<tr>
<td>( k )</td>
<td>7</td>
<td>A higher value of ( k ) indicates that the compressive stress within the network is stronger. Consequently, the bed layer is harder to compress</td>
<td>[8]</td>
</tr>
<tr>
<td>( \sigma_0 )</td>
<td>50 Pa</td>
<td>A positive number carrying the unit of pressure. It has the same effect as changing ( k ), but to different extent</td>
<td>[14]</td>
</tr>
</tbody>
</table>

### 4.3. Model Validation

By using the value of parameters presented above, simulation is carried out to validate the accuracy of the model. Data of feed flow rate, feed solids concentration and underflow removal rate from industry are input to the model, and the result is shown in Figure 5.

It is observed that for most of the time, the simulated results can follow the plant data closely. These findings show that
the mathematical model used to describe the dynamics of the paste thickener is reasonably accurate and (2.) the values of parameters used in the study can be used to describe the coal characteristic satisfactorily. The validated model and knowledge about the values of parameters will facilitate the development of a control system in the future.

CONCLUSIONS AND RECOMMENDATIONS

In this article, the sedimentation-consolidation model is discretized to facilitate the analysis of the system. Findings show that a change in underflow withdrawal rate affects the system more rapidly as compared to a change in inlet flow solids. The open-loop simulation shows the model output is in a good agreement with plant data. The results are expected to be improved by using a varying parameter that changes over time. This can happen in the form of coal type changes which occurs frequently in real world coal preparations. One way to improve the model accuracy is to implement an online parameter estimator (e.g. a Kalman filter) based on measured data, which is one possibility of the extension of this work.

Different dynamics shown by the system lead to a complicated and sophisticated system. The formation of bed layer has found to affect the dynamical system considerably. This shows a need to design a control system to ensure a smooth operation of paste thickeners. Therefore, it is interesting to go one step further to develop a modern control approach, particularly Model Predictive Control (MPC) that is anticipated to improve the operation of paste thickeners.

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