GRAVITY FLOW OF GRANULAR SOLIDS THROUGH ORIFICES OF DIFFERENT SHAPES IN A HOPPER

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ABSTRACT
Numerical investigation of gravity flow of granular solids in a hopper through orifices of different shapes has been conducted using the discrete element method. The simulations are validated by comparing with the experimental results for rapeseed cases. It is observed that with orifices of the same area the flow rate decreases in the order of circle, square, rectangle and triangle, which is similar to the experimental outcome. A formula for flow rate reflecting the effect of orifice shape is obtained, and compared with the Beverloo’s equation. The reasons behind the influence of orifice shape on flow rate are further studied by considering different internal physical properties of the granular system in the arching zone configurations of the hopper. The velocity field of particles and inter-particle forces have been analysed. It is found that these properties near the orifice area vary for different orifice shape, which can be used to explain the influence of orifice shape on particle flow rate.

INTRODUCTION
Flow characteristic of granular solids is a significant factor in bulk materials handling area. There have been a great number of studies in this field, especially on the flow rate (Beverloo et al., 1961; McDougall and Evans, 1965; Langston et al., 1995; Zhu and Yu, 2005), arch formation (Richmond and Gardner, 1962; Shinohara et al., 1968), and flow pattern in the particle layer (Brown and Richards, 1960). Flowing characteristics are significantly influenced by the orifice area settings (Huang et al., 2006). The effect of orifice shape has been analysed experimentally by, for example, Fowler and Glastonbury (1958) and Beverloo et al. (1961). The flow rate equations for dry particles have been obtained based on the experimental results. However, to our knowledge, the particle scale information has not been considered to explain on the effect of different orifice shapes on flow rate. Much work has been carried out to demonstrate that the particle scale information is essential to understand the mechanism of particle flow behaviour (Zhu et al., 2008). In this work, the effects of orifice shape of a hopper on the flow rate and the microscopic properties of the hopper flow are considered by use of the discrete element method (DEM).
COMPUTATIONAL MODELS

Simulation method
The DEM is employed to generate microscopic quantities for the present micromechanical analysis. In the DEM simulation the motion of every particle, which can undergo translational and rotational motions, is described by Newton's laws of motion. These equations, which are based on the forces and torques originated from its interaction with neighbouring particles, can be solved numerically. Thus, the trajectories, velocities and transient forces of all the particles in a system can be determined. The inter-particle forces involved are contact forces between particles, which are related to the relative displacements of the particles at contact. The torque acting on a particle includes two components: one is generated by the tangential force and causes the particle to rotate, and another commonly known as the rolling friction torque, is generated by asymmetric normal forces and slows down the relative rotation between particles. The equations used to calculate the particle-particle interaction forces and torques can be found in our recent review (Zhu et al., 2007). In this study, the commercial software EDEM™ based on this method has been used to simulate the particle flows.

Simulation parameters and conditions
The geometry of the hopper considered is cylindrical in shape, of diameter of 5 cm and with a circular, square, rectangle (1:2) or triangle (1:1:1) shaped orifice of varying area of 0.785, 1.226 and 1.766 cm$^2$ at the centre of its flat bottom, as shown in Fig. 1. The initial particle bed height is 10 cm.

![Fig. 1: Geometry of hopper model](image)

To select appropriate simulation parameters the calibration process for DEM simulation is carried out. The experimental case of Beverloo (1961) is considered to compare with the benchmark simulation case. Spherical particles are considered in this work. The particles have a similar shape factor like rapeseed used in the experiments. The particle diameter is 0.15 cm, particle density is 1.12 (g/cm$^3$), sliding friction coefficient is 0.6 for particle-particle (p-p) and 0.3 for particle-wall (p-w), rolling friction coefficient is 0.001 cm for p-p and p-w, shear modulus (p or w) is 1e7 Pa, Poisson ratio (p or w) is 0.25 and coefficient of restitution (normal or tangential, p-p or p-w) is 0.5. From Tab. 1 we can see that most of the materials and bulk properties are similar in both physical experiment and simulation cases. Angle of repose is also similar for both cases and
adjusted in the DEM simulation by adjusting rolling friction and static friction. Angle of repose is measured using the similar setup like the experiment of Beverloo (1961).

<table>
<thead>
<tr>
<th>Materials</th>
<th>Bulk density $\rho_B$ (g/cm$^3$)</th>
<th>Void fraction (%)</th>
<th>Average screen size d (cm)</th>
<th>Shape factor</th>
<th>Angle of repose, $\phi_s$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rapeseed</td>
<td>Experiment</td>
<td>0.67</td>
<td>40.2</td>
<td>0.17</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>Simulation</td>
<td>0.64</td>
<td>43.1</td>
<td>0.15</td>
<td>1</td>
</tr>
</tbody>
</table>

Tab.1: Physical properties of the materials used in the flow experiments

RESULTS AND DISCUSSIONS

The particle discharge profiles through different orifice shapes with the same area (0.785 cm$^2$) is shown in Fig. 2. The number of discharged particles in the time interval of 0.1 second is plotted to show the variation the discharge profile with time. The variation of discharged particle number has a similar trend for circle and square cases and the instantaneous discharged particle number for both cases is higher compared to triangle and rectangle cases. In the cases of triangle and rectangle the number of discharged particles is consistently low and in the case of triangle the number of discharged particles is the lowest. The instants of highest and lowest number of discharged particles in a hopper discharge profile indicate the arching formation and collapse near the orifice. During the arching formation particles near the hopper orifice slow down and hopper discharge is lower. Although the frequency of the variation has not been clearly determined, but it is clear that the variation in the discharge profile is higher in the cases of triangle and rectangle.

Fig. 2: Particle discharge profile in different types of orifice (data is taken at time interval of 0.1 second)
A set of numerical experiments by DEM is carried out to analyse the effect of orifice shape on the flow rate of hopper. The minimum orifice diameter is at least five times larger than the particle diameter. In Fig. 3 the effect of orifice area on mass flow rate in physical and numerical experiments is shown. It can be seen that the flow rate increases with the increase of orifice area for both physical and numerical experiments. For the same orifice area the flow rate also decreases in the order of circle, square, rectangle and triangle shapes for both experiments. For the higher orifice area the difference in the flow rate due to shape factor is more evident. The flow of rapeseed through orifices of various shapes from simulation results can be calculated via following equations:

\[ W = 45.0 \, \rho_b \, A^* \sqrt{g \, D^*_h} \]  \hspace{1cm} (Circle)  \\
\[ W = 46.0 \, \rho_b \, A^* \sqrt{g \, D^*_h} \]  \hspace{1cm} (Square)  \\
\[ W = 50.0 \, \rho_b \, A^* \sqrt{g \, D^*_h} \]  \hspace{1cm} (Rectangle)  \\
\[ W = 55.0 \, \rho_b \, A^* \sqrt{g \, D^*_h} \]  \hspace{1cm} (Triangle)  \\

where, \( \rho_b \) is bulk density of packing, \( D^*_h \) is effective hydraulic diameter and \( A^* \) is effective orifice area.

From the flow rate equations it can be seen that the coefficients for circle, square, rectangle and triangle are 45, 46, 50 and 55, respectively. In the experiment coefficients for circular, square, rectangle and triangle cases are 45, 46.6, 44.4 and 45.7, respectively (Beverloo et al., 1961). The values of the coefficients are almost similar in cases of circle and square, but slightly different in cases of rectangle and triangle. The difference in the numerical and physical experiments may be caused by the fact that slightly different shape factor and particle size distributions have been adopted in simulations compare to physical experiments. This also indicates that shape factor and particle size distributions may affect particle flow rate equation more in rectangle and triangle cases than circle and square cases.
Fig. 4 shows total particle flow velocity structure after 5 seconds of particles discharge through different orifice shapes with the same area (0.785 cm\(^2\)). It is clear that the structure varies for different orifice shapes. The exact shape of the arching zone is difficult to determine and in these cases particles with velocity less than 0.12 m/s and higher than 0.06 m/s have been considered to be in the arching zone. Stagnant zone and the arching zone near the orifice are observed for all considered cases. In circle case the particles are more mobile in the arching zone compare to other cases. In this case the size of the stagnant zone is smaller and the arching zone is in a higher position from the orifice. The arching zone for square and triangle is smaller than that of circle. In the rectangle case the arching zone is wider but shorter in height.

(a) Circle                  (b) Square             (c) Triangle         (d) Rectangle

Fig. 4: Particle flow velocity structure inside a hopper for different orifice shapes

To understand the behaviour of particle flow in the hopper more clearly, the particle scale information near the orifice area is checked. A cylindrical volume of 1cm radius and 2 cm height is considered in orifice area to cover the arching zone. Before particle discharge the average interparticle contact or compressive force between particles of packing in the orifice area is high and the value is 427 dyne for all the cases. After the orifice is opened the particles flow out of the hopper through the orifice and the particles in that zone create arching configurations. Different arching zones are formed for different orifice shapes. In the arching zone particle flow is dense and mostly collisional. The profile of compressive force between the particles in the arching zone varies in different cases. The average compressive force for each particle is calculated over a time period and for different orifice shapes the average compressive force profiles are shown in Fig. 5. The compressive force in the arching zone is closely linked with the motion characteristic of the particles.

The collisions between particles are also recorded when they collide and the total collision numbers in the arching zone are used to show the variation for different cases. In Fig. 6 and Fig. 7, we can see the profiles of total collision number between particles in the arching zone and the average number of particle contacts inside the hopper for different orifice shapes during particle discharge. It can be seen that the collision number decreases in the order of circle, square, rectangle and triangle cases. Average particle contact number is related to particle coordination number in particle packing. Fig. 7 shows that average contact number per particle increases in the order of circle, square, rectangle and triangle cases.
Fig. 5: Average compressive force between particles during discharge

Fig. 6: Number of collisions with discharge time

Fig. 7: Number of total contacts between particle in the hopper with discharge time
Tab. 2 lists the average particle velocity, average compressive force between particles, total collision number between particles per 0.1 second and average contact number per particle after 5 seconds of discharge. It can be seen that the average particle velocity is decreasing in the order of circle, square, rectangle and triangle case. The average compressive force and average collision number between particles per 0.1 second in the arching zone are found to be related to the average velocity. For the higher average velocity, the average compressive force is lower and the average collision number is higher. Average contact number per particle in the hopper is also indicating that coordination number is higher for higher average particle velocity.

<table>
<thead>
<tr>
<th>Orifice type</th>
<th>Average velocity (cm/s)</th>
<th>Average compressive force (dyne)</th>
<th>Average collision number</th>
<th>Average contact number per particle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td>1.36</td>
<td>92.1</td>
<td>912706</td>
<td>2.9263</td>
</tr>
<tr>
<td>Square</td>
<td>1.22</td>
<td>92.9</td>
<td>863995</td>
<td>2.9213</td>
</tr>
<tr>
<td>Rectangle</td>
<td>1.08</td>
<td>93.2</td>
<td>803176</td>
<td>2.9005</td>
</tr>
<tr>
<td>Triangle</td>
<td>1.05</td>
<td>93.7</td>
<td>796962</td>
<td>2.9004</td>
</tr>
</tbody>
</table>

Tab. 2: Average particle velocity, compressive force, collision number and contact number for different orifices (data is taken at time interval of 0.1 second)

The arches of particles are formed and collapsed in the arching zone repeatedly during particle discharge. The collapse and formation of the arching are present in all the cases. Fig. 8 shows the particle velocity and contact vector diagram at two instants reflecting the arching collapse (higher discharge) and formation (lower discharge) for the four orifice cases. It can be seen that during arching collapse the particles are more mobile and in less contacts in that zone compare to arching formation instant. In the cases of rectangle and triangle there are fewer particles with higher velocities and higher number of contact between particles compare to circle and square cases. The frequency of arching formation and collapse also varies for all the cases. The results indicate that the orifice shape is an important factor to affect the arching formation and collapse phenomena.
CONCLUSIONS

Orifice shape is an important factor for the hopper flow rate and flow patterns inside the hopper. Our simulation studies confirmed the findings by experiments from different researchers on flow rate. For the same orifice area the flow rate decreases in the order of circle, square, rectangle and triangle shapes. Furthermore, the analysis of the particle scale information indicates that flow structures, including stagnant zone and arching zone configurations vary significantly for different orifice shapes. The average particle collision number, particle contacts number and particle velocity in the that area decrease, while the compressive force increases in the order of circle, square, rectangle and triangle orifice cases. The particle arching configuration near the orifice and the frequency of arching formation and collapse in the arching zone affect the flow characteristics and flow rate.

REFERENCES


