MODELING AND NUMERICAL SIMULATION OF LIQUID-SOLID CIRCULATING FLUIDIZED BED SYSTEMS

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ABSTRACT
A novel liquid-solid circulating fluidized bed (LSCFB) is modelled for protein recovery from feed broth. A typical LSCFB system consists of two individual columns (downer and riser), integrating two different operations (adsorption and desorption respectively) simultaneously with continuous adsorbent circulation between these columns. A general purpose, extensible, and dynamic model was written based on the tanks-in-series framework. The model allows adjusting the degree of back-mixing in each phase (solid and liquid) for both riser and downer. The model is validated with previously published data on extraction of bovine serum albumin (BSA) as model protein. Detailed dynamic analysis is performed on the protein extraction operation. The interaction between the riser and the downer are captured. Parametric studies on protein recovery in LSCFB system are carried out using the validated model to better understand the system behaviour. Simulation results have shown that both production rate and overall recovery increase with an increase in solid circulation rate and feed flow. With an increase in the entering feed concentration, the rate of production increases, but the overall recovery decreases. The model is flexible and can use various forms of ion exchange kinetics and can simulate different hydrodynamic behaviours. It is useful to gain insight into protein recovery process. The general nature of the model makes it useful to study other protein recovery operations for plant and animal proteins. It can also be useful for further multi-objective optimization studies to optimize the LSCFB system.

INTRODUCTION
Liquid-solid circulating fluidized bed (LSCFB) systems are rapidly being applied in biochemical separation technology (Nakhla et al. 2004, Patel et al. 2006). Typical LSCFB systems consist of two parts viz. a downer and a riser, integrating principal reactions or adsorption processes with continuous circulation of particles between these two. Typically, the downer is used for reaction or adsorption processes to provide a longer residence time. Whereas, the riser, with its higher liquid velocity and excellent plug flow characteristics, is used for the fast regeneration of desorption of adsorbents. Lan et al. (2000, 2002a) introduced the concept of LSCFB for adsorption processes. They studied the effect of operating conditions on hydrodynamics of LSCFB and successfully developed LSCFB systems for continuous recovery of bovine serum albumin (BSA) and whey proteins from unclarified broths using Diaion HPA25® anion ion exchangers. Patel et al. (2006) developed a LSCFB...
system with anoxic and aerobic beds for simultaneous removal of carbon, nitrogen and phosphorus from municipal wastewater. While these studies have proven that such LSCFB adsorption have potential applications, these systems are still poorly understood. In addition to all the benefits of fluidization, including low and stable pressure drops across the fluidized beds, this technology has attracted attention for its enhanced mass and heat transfer, reduced backmixing, and easy handling of particles of mixed sizes and densities lead to a much more effective processing (Zhu et al. 2000). Besides, the riser-downer configuration of LSCFB makes it possible to have continuous adsorption processes with adsorption and desorption conducted simultaneously, further enhancing the efficiency and reduction in equipment size (Atta et al. 2009; Lan et al. 2002a). Overall, LSCFB systems have advantages of economy; nevertheless their success is strongly dependent on better understanding of the LSCFB dynamics.

In this paper, the application of the LSCFB system for continuous protein recovery was studied. A general purpose, extensible, and dynamic mathematical model based on the compartments-in-series approach was established. The model allows for adjustment of the backmixing degree in each phase (solid and liquid) for both riser and downer thus providing a flexibility to match the residence time distribution of industrial systems. The model predictions were validated using the experimental data on the recovery of BSA onto Diaion HPA25® in a LSCFB system (Lan et al. 2002b). The validated model was then used to study the effects of various parameters on LSCFB performance.

**PROTEIN RECOVERY USING LSCFB**

A schematic diagram of the liquid-solid fluidized bed (LSCFB) system is shown in Fig. 1 (Mazumder et al. 2009). The system is used for continuous recovery of bovine serum albumin (BSA) to polymeric adsorbents Diaion HPA25®. The continuous protein adsorption is conducted with downer as the adsorption vessel, while, the riser is used for adsorbent regeneration. The downer operates in conventional fluidization regime. The liquid velocity is kept below the particle terminal settling velocity but enough to fluidize the particles. The feed stream is injected through a distributor into the bottom of the downer, and the solids move down countercurrent to the rising feed stream. The particles travel from downer to the bottom of riser through solids feed pipe. The riser operates in fast fluidization regime. The primary and auxiliary liquid streams are injected into the bottom of the riser. The primary liquid stream is used to transport the particles, whereas, the auxiliary liquid stream provides additional mixing in the distributor region. The riser operates at total liquid velocity higher than the terminal settling velocity of the particles. Liquid and the solid move concurrently along to the top of the riser where they are separated by a liquid-solid separator. The particles are then transferred to the top of the downer via the solids return pipe. Tab. 1 shows the specifications of LSCFB studied in this work as well as the properties of the adsorbent.

**MODEL DEVELOPMENT**

The performance of the LSCFB closely depends on the hydrodynamics and how the different phases (solids and liquid) are distributed in the downer and riser. Therefore, a model that allows to describe the hydrodynamics in a flexible manner is desirable. The compartments-in-series framework allows to adjust the degree of backmixing in each of the phase independently. Additionally it allows flexibility in adjusting the residence time distribution of different phases. Therefore, it was chosen as the basis of the model.
A schematic representation of the LSCFB model is shown in Fig. 2. The mixing patterns in downer and riser can be represented by a series of ideally mixed tanks. The tank-in-series model is chosen because it not only allows easy integration with the kinetics model, but also offers a straightforward comparison of the reactor performance with that of a plug flow reactor as reported previously (Lan et al. (2000) and Mazumder et al., 2009). Each column in the system is divided into two series of ideally mixed stirred tanks; one corresponding to liquid phase, while the other to solid phase. Diaion HPA25® particles are referred to as the solid phase in the diagram. In the LSCFB, the entrained solid particles do not flow convectively through the downer and riser column in contrast to the liquid flows. Subsequently, the mixing in solid phase is relatively extensive than that in the liquid phase. Thus, the solids phase can be represented by fewer tanks than the liquid phase. In the current model, the solid phase is constituted by M equal size ideally mixed stirred tanks, arranged in series, and each solid tank is then further subdivided into a series of N ideally mixed subtanks of liquid phase.

The following assumptions are made in developing the model:

- Adsorption rate is limited by intra-particle diffusional resistance and mass transfer resistance in the laminar fluid boundary layer surrounding an individual particle.
- Surface adsorption is instantaneous and thus a local equilibrium is established at the particle surface between protein concentrations in the two phases. The equilibrium behaviour of protein adsorption is well-described by the Langmuir isotherm.
- Adsorbent particles are spherical and uniform in size with a mean diameter of $d_p$. These particles are relatively immobile.
- Protein concentration in liquid solution of the dilute region in downer is very low, thus adsorption in these areas are negligible compared to that of the dense region.
- Particle concentration and solids holdup are uniformly distributed across the system.
- Effects of liquid phase axial dispersion and solid backmixing in each tank are negligible.
- Thermal effects are negligible, i.e. the system operates isothermally.

Based on these assumptions, the transient model equations for the downer and the riser are developed. The equations are given in Tabs. 2 and 3. In order to close the model equations, information on various hydrodynamic parameters is required. The framework is flexible in selecting correlations for these parameters. The selection of correlations used in the paper is given in Tab. 4.

**NUMERICAL SIMULATION**

The set of governing equations listed in Tabs. 2 and 3 yield a initial value problem with coupled ordinary differential equations (ODEs) for simulating the system performance. MATLAB® was used to simulate the system of ODEs. Two model parameters specified at the outset of the simulation were, the number of tanks-in-series in each phases used to assemble the two entrained columns. Mixing behaviour in particles is considerably extensive than liquid phase, thus the former is represented by fewer tanks than the latter. Initially, the liquid phase superficial velocities and solid circulation rate were given, and the particle superficial velocities were calculated. In this study, MATLAB® built-in solver ODE45 was used to solve the system ODEs to find the concentration profile along the distributor and upper dilute regions of riser. The concentration in solid particles at the top of the riser ($q_{er}$) was used as the new value of $q_{od}$ as no adsorption occurred inside solids feed pipe. Subsequently, the second cycle
commenced with the calculated values of $C_{ed}$ and $q_{od}$, and the set of system ODEs were solved repeatedly in the same manner. Based on this iteration, the solver iterated over the next time step until $C_{ed}$ has reached a state of convergence. The model parameters used in the simulation are given in Tab. 5

**MODEL VALIDATION**

The model results were compared against the experimental data for liquid phase BSA protein concentration profile reported in the literature (Lan et al. 2000, 2002b). Figs. 3 and Fig. 4 show the experimentally obtained BSA concentration profile along the downer with variations in solids circulation rate ($G_s$) and superficial liquid velocities in downer ($U_{ld}$) respectively. The other system parameters were kept constant (listed in Tab. 6). As can be seen, the magnitude and trends of the model predictions are in reasonably good agreement with the experimental data over almost all the range. One significant difference is, the protein concentration profiles anticipated are slightly higher than the experimentally reported values at the lower part of the downer ($ih_{cap}/h_{cap}<0.3$). The reason for this difference is probably due to the rapid initial solids acceleration upon entering the system, because of fluid drag forces interaction with other particles in the entrance regions close to the distributors, and then more gradually further down the columns. At the same time, the flow structure developed accordingly from a non-uniform distribution into a more uniform distribution. Rapid initial solids acceleration resulted in higher tendencies of solids backmixing in operating regions near the liquid distributors. While, this can be accommodated in the present modelling framework by altering the number of tanks in the section near to the solids entrance, no special effort was made to adjust it as detailed residence time distribution profiles were not available.

**RESULTS AND DISCUSSION**

With the model validated, parametric sensitivity analysis of some key parameters was conducted to obtain a better understanding of mass transfer and hydrodynamics in the system. At a given inventory of solid particles, the simultaneous adsorption and desorption behaviour of the protein at the steady state is dependent mainly on: superficial liquid velocities in the riser ($U_{lr}$) and downer ($U_{ld}$), the solids circulation rate ($G_s$), and the feed protein concentration ($C_{od}$). These parameters were varied to estimate the concentration profiles along the riser and the downer as well as the protein production rate and overall protein recovery.

**Effect of Solids Circulation Rate**

The effect of $G_s$ on the BSA concentration profiles in the downer is shown in Fig. 5. For a given total liquid flowrate, the auxiliary liquid velocity was adjusted in order to yield variations of the solids circulation rate ($G_s$). The BSA concentration in the raffinate exiting the downer ($C_{ed}$) decreases with an increase in $G_s$, resulting in a decreasing concentration gradient when equilibrium is reached. At constant superficial liquid velocity, the bed height of the dense region ($h_{cap}$) increases with increase in $G_s$, resulting in a higher dynamic adsorption capacity of the downer, as more interfacial contact area becomes available. Furthermore, higher $G_s$ increased the liquid-solid slip velocities. Hence high liquid-solid interfacial contact efficiency is expected for improved mass transfer coefficient ($K_{La}$) in the downer dense region. At the same time, the solids holdup in the dense region ($\varepsilon_{cap}$) decreases with $G_s$, as higher auxiliary liquid flowrates yields higher particle velocities, and the solid phase residence time in the downer is reduced. Therefore, a steeper concentration profile is observed at higher $G_s$. 
Fig. 6 shows the predicted liquid phase protein concentration profiles in the riser at different solids circulation rate ($G_s$) keeping the other parameters at the base case values. The solids phase is dense in the distributor region and relatively more dilute further down the riser. Nonetheless, at a given liquid velocity, the non-uniformity of solids distribution increases with the $G_s$, leading to a slight drop in the solids holdup gradient and thus reducing the extent of protein desorption in the riser. The characteristic of the flow structure in the riser suggests that the liquid-solid mixing along the length of the riser is likely to be non-uniform near the distributor but becomes uniform further down the column. Consequently, the protein production rate ($P$) increases from 32.26 to 41.63 g/h and the overall protein recovery ($R$) increases from 66.08 to 85.27% with the increase in $G_s$ from 1.06 to 1.42 kg/m$^2$/s.

**Effect of Superficial Liquid Velocity in Downer**

Figs. 7 and 8 illustrate the protein concentration profiles of the downer and riser liquid phase when the superficial liquid velocity in the downer ($U_{ld}$) is increased from 1.06 to 1.42 kg/m$^2$/s respectively. The values of BSA concentration in the raffinate stream ($C_{ed}$) were found to increase steeply with increasing $U_{ld}$, suggesting more protein is lost at higher $U_{ld}$. This could be explained by the a shorter liquid phase residence time in downer due to increasing $U_{ld}$ and therefore reduced time for protein adsorption and higher $C_{ed}$ from the raffinate stream. Increasing $U_{ld}$ also affects the height of the dense region ($h_{d^{eff}}$). The $h_{d^{eff}}$ increases with increase in the $U_{ld}$ studied, and thereby the solids holdup is reduced in the bed. The mass transfer coefficient ($K_La$) increases slightly with increasing $U_{ld}$. But the effect of changes in mass transfer is relatively small compared to those in liquid phase residence time and solids holdup. Since the protein loading rate and downer dense region height increase with $U_{ld}$, higher amount of adsorbed protein are carried to the riser in the solid particles exiting the downer ($q_{ed}$). This also yields higher production rates and reduced protein recovery as significant amount of BSA lost into the raffinate stream.

**Effects of Superficial Liquid Velocity in Riser**

Figs. 9 and 10 show the variations of liquid phase protein concentration profile in the downer and riser with superficial liquid velocity in the riser ($U_{lr}$). For a given solids circulation rate, It is observed that the higher the $U_{lr}$, the smaller the protein concentration in the extract ($C_{er}$) is. With increasing liquid velocity in the riser, the drag force exerted by the upward flowing liquid increases, this decreases the residence time of the solid phase resulting into reduced protein desorption capacity. The height of the dense region of downer ($h_{d^{eff}}$) increases uniformly with the solids holdup reduction in the riser ($\varepsilon_{sr}$), since more particles are transferred to downer because of higher $U_{lr}$ in the riser. Nonetheless, increase of $h_{d^{eff}}$ is compensated by decrease of adsorption capacity in the downer due to reduced desorption capacity as protein concentration in the regenerated particles ($q_{od}$) increases. However, the effect of changes in $q_{od}$ to the system mass transfer is relatively small owing to the constant solids holdup in the downer dense region ($\varepsilon_{sd}$). The increase in the superficial liquid velocity in riser reduces both the adsorption and desorption capacities of the system reducing both the protein production rate and the protein recovery dramatically with the increase in $U_{lr}$.

**CONCLUSIONS**

A general purpose, extensible, and dynamic theoretical compartmental model based upon a tanks-in-series framework incorporating the equilibrium and hydrodynamics of liquids and solid particles has been developed for continuous protein recovery in liquid-solid circulating
fluidized bed (LSCFB) systems. The model was used to simulate the recovery of aqueous bovine serum albumin (BSA) solution onto Diaion HPA25® particles. The model allows adjusting for the degree of back-mixing in each phase for the riser and the downer, while make possible easy integration with the kinetics model and offer a straightforward comparison of the reactor performance with that of a plug flow reactor. The simulated results compare well with the experimental results obtained from the laboratory-scale BSA recovery. A systematic study of the effect of several key operating parameters was performed. The analysis revealed that both the BSA production rate and recovery increase with increasing solids circulation rate, while both of them decrease with increasing superficial liquid velocity in the riser. With the increase in superficial liquid velocity in the downer and feed BSA concentration, the rate of BSA production increases, but the overall recovery decreases. The computational model derived in this paper is flexible and can use different forms of ion exchange kinetics and can simulate different hydrodynamic behaviour in order to gain insight into protein recovery processes. The very nature of the model makes it a useful tool in learning other protein recovery operations for plant and animal proteins. It can also be utilized for further multi-objective optimization studies to optimize LSCFB systems.

REFERENCES
Fig. 2: Schematic representation of the LSCFB model (a) the downer, (b) the riser

Fig. 3: Liquid phase protein concentration profile in the downer under different solids circulation rate ($C_{od}$=2kg/m$^3$, $U_{ld}$=0.6mm/s, $U_{lr}$=11.3mm/s, S=3kg)

Fig. 4: Liquid phase protein concentration profile in downer under different superficial liquid velocity ($C_{od}$=2kg/m$^3$, $G_r$=1.24kg/m$^2$s, $U_{lr}$=11.3mm/s, S=3kg)
Fig. 5: Liquid phase protein concentration profile in the downer under different solids circulation rate, $G_s$ ($C_{os}=2\text{kg/m}^3$, $U_{ld}=0.6\text{mm/s}$, $U_{lr}=11.3\text{mm/s}$, $S=3\text{kg}$)

Fig. 6: Liquid phase protein concentration profile in the riser under different solids circulation rate, $G_s$ ($C_{os}=2\text{kg/m}^3$, $U_{ld}=0.6\text{mm/s}$, $U_{lr}=11.3\text{mm/s}$, $S=3\text{kg}$)

Fig. 7: Liquid phase protein conc. profile in downer under different superficial liquid velocity in the downer, $U_{ld}$ ($C_{os}=2\text{kg/m}^3$, $G_s=1.24\text{kg/m}^2/\text{s}$, $U_{lr}=11.3\text{mm/s}$, $S=3\text{kg}$)

Fig. 8: Liquid phase protein conc. Profile in riser under different superficial liquid velocity in the downer, $U_{ld}$ ($C_{os}=2\text{kg/m}^3$, $G_s=1.24\text{kg/m}^2/\text{s}$, $U_{lr}=11.3\text{mm/s}$, $S=3\text{kg}$)

Fig. 9: Liquid phase protein conc. profile in the downer under different superficial liquid velocity in the riser, $U_{lr}$ ($C_{os}=2\text{kg/m}^3$, $G_s=1.24\text{kg/m}^2/\text{s}$, $U_{ld}=0.60\text{mm/s}$, $S=3\text{kg}$)

Fig. 10: Liquid phase protein conc. profile in the riser under different superficial liquid velocity in the riser, $U_{lr}$ ($C_{os}=2\text{kg/m}^3$, $G_s=1.24\text{kg/m}^2/\text{s}$, $U_{ld}=0.60\text{mm/s}$, $S=3\text{kg}$)
### TABLES

#### Tab. 1: Specifications of the LSCFB system

<table>
<thead>
<tr>
<th>Downer and riser (LSCFB system)</th>
<th>Adsorbent particle (Diaion HPA25&lt;sup&gt;®&lt;/sup&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downer bed height, $H_d$</td>
<td>2.5 m</td>
</tr>
<tr>
<td>Downer bed diameter, $D_d$</td>
<td>120 mm</td>
</tr>
<tr>
<td>Riser bed height, $H_r$</td>
<td>3.0 m</td>
</tr>
<tr>
<td>Riser bed diameter, $D_r$</td>
<td>38 mm</td>
</tr>
<tr>
<td>Riser distributor region height, $H_{r1}$</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Separator height, $H_c$</td>
<td>177 mm</td>
</tr>
<tr>
<td>Separator upper section diameter, $D_c$</td>
<td>120 mm</td>
</tr>
<tr>
<td>Separator lower section diameter, $d_c$</td>
<td>35 mm</td>
</tr>
<tr>
<td>Solids return pipe length, $L_{re}$</td>
<td>500 mm</td>
</tr>
<tr>
<td>Solids return pipe diameter, $D_{re}$</td>
<td>35 mm</td>
</tr>
<tr>
<td>Solids feed pipe length, $L_{fe}$</td>
<td>800 mm</td>
</tr>
<tr>
<td>Solids feed pipe diameter, $D_{fe}$</td>
<td>35 mm</td>
</tr>
</tbody>
</table>

#### Tab. 2: Downer model equations

**Liquid phase mass balance equations**

\[
\frac{dC_{i,d}}{dt} = \frac{U_{id}(C_{i-1,d} - C_{i,d})}{h_{id}\epsilon_d} - \frac{K_i a (1 - \epsilon_d) (C_{i,d} - C_{eq})}{\epsilon_d} \quad \text{for } 1 < i < MN
\]

\[
\frac{dC_{i,d}}{dt} = \frac{U_{id}(C_{1,d} - C_{i,d})}{h_{id}\epsilon_d} - \frac{K_i a (1 - \epsilon_d) (C_{i,d} - C_{eq})}{\epsilon_d} \quad \text{for } i = 1
\]

\[
\frac{dC_{MN,d}}{dt} = \frac{U_{id}(C_{MN-1,d} - C_{MN,d})}{h_{id}\epsilon_d} - \frac{K_i a (1 - \epsilon_d) (C_{MN,d} - C_{eq})}{\epsilon_d} \quad \text{for } i = MN
\]

**Solid phase mass balance equations**

\[
\frac{dq_{m,d}}{dt} = \frac{U_{sd}(q_{m+1,d} - q_{m,d})}{h_{sd}(1 - \epsilon_d)} + \sum_{i=\text{max}(1)}^{\text{min}(N+1)} \frac{K_i a (1 - \epsilon_d) (C_{i,d} - C_{eq})}{\epsilon_d} \quad \text{for } 1 < m < M
\]

\[
\frac{dq_{1,d}}{dt} = \frac{U_{sd}(q_{2,d} - q_{1,d})}{h_{sd}(1 - \epsilon_d)} + \sum_{i=1}^{N} \frac{K_i a (1 - \epsilon_d) (C_{i,d} - C_{eq})}{\epsilon_d} \quad \text{for } m = 1
\]

\[
\frac{dq_{M,d}}{dt} = \frac{U_{sd}(q_{M,d} - q_{M-1,d})}{h_{sd}(1 - \epsilon_d)} + \sum_{i=\text{max}(N+1)}^{\text{max}(M-1)} \frac{K_i a (1 - \epsilon_d) (C_{i,d} - C_{eq})}{\epsilon_d} \quad \text{for } m = M
\]

**Lumped mass transfer coefficient**

\[
k_f = \frac{D_m}{d_p}[2 + 1.03 (\epsilon_{sd} Re_p)^{0.5} (Sc)^{0.33}]
\]
\[ K_L = \psi k_f \]

**Langmuir isotherm**

\[ C_{eq} = \frac{K_d q_d}{q_m - q_d} \]

---

**Liquid phase mass balance equations**

\[
\frac{dC_{i,r}}{dt} = \frac{U_f (C_{i-1,r} - C_{i,r})}{h_f \varepsilon_r} + \frac{k_r q_{m,r} (1 - \varepsilon_r)}{\varepsilon_r} \quad \text{for } 1 < i < MN
\]

\[
\frac{dC_{1,r}}{dt} = \frac{U_f (C_{1,in} - C_{1,r})}{h_f \varepsilon_r} + \frac{k_r q_{1,r} (1 - \varepsilon_r)}{\varepsilon_r} \quad \text{for } i = 1
\]

\[
\frac{dC_{MN,r}}{dt} = \frac{U_f (C_{MN-1,r} - C_{MN,r})}{h_f \varepsilon_r} + \frac{k_r q_{MN,r} (1 - \varepsilon_r)}{\varepsilon_r} \quad \text{for } i = MN
\]

**Solid phase mass balance equations**

\[
\frac{dq_{m,r}}{dt} = \frac{U_f (q_{m-1,r} - q_{m,r})}{h_f (1 - \varepsilon_r)} - \sum_{i=(m-1)N+1}^{(m-1)N+N} \frac{k_r q_{m,r} (1 - \varepsilon_r)}{\varepsilon_r} \quad \text{for } 1 < m < M
\]

\[
\frac{dq_{1,r}}{dt} = \frac{U_f (q_{1,in} - q_{1,r})}{h_f (1 - \varepsilon_r)} - \sum_{i=1}^{N} \frac{k_r q_{1,r} (1 - \varepsilon_r)}{\varepsilon_r} \quad \text{for } m = 1
\]

\[
\frac{dq_{MN,r}}{dt} = \frac{U_f (q_{MN-1,r} - q_{MN,r})}{h_f (1 - \varepsilon_r)} - \sum_{i=(M-1)N+1}^{MN} \frac{k_r q_{MN,r} (1 - \varepsilon_r)}{\varepsilon_r} \quad \text{for } m = M
\]

---

**Tab. 3: Riser model equations**

**Tab. 4: Hydrodynamic correlations used in the model**

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Equation</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downer effective bed height, ( h_{d,eff} )</td>
<td>( h_{d,eff} = \frac{S/ \rho_d - (h_{r1} \varepsilon_{r1} + h_{r2} \varepsilon_{r2}) A_r - V_p (1 - \varepsilon_p)}{\varepsilon_{sd} A_d} )</td>
<td>Mazumder et al. 2009, 114; Zheng and Zhu 2000, 146</td>
</tr>
<tr>
<td>Solid volume in LSCFB sections, ( V_p )</td>
<td>( V_p (1 - \varepsilon_p) = \frac{4}{3} \varepsilon_f \varepsilon_c A_c - L_{re} A_r (1 - \varepsilon_{re}) - L_{fe} A_f e (1 - \varepsilon_{fe}) )</td>
<td>Mazumder et al. 2009, 114; Zheng and Zhu 2000, 146</td>
</tr>
<tr>
<td>Downer dense region voidage, ( \varepsilon_{d1} )</td>
<td>( U_{id} + U_{sd} \frac{\varepsilon_d}{1 - \varepsilon_d} = U_i \varepsilon_{d1} )</td>
<td>Lan et al. 2000, 861; Kwaau 1992</td>
</tr>
<tr>
<td>Riser distributor region voidage, ( \varepsilon_{r1} )</td>
<td>( U_{ir} - U_{sr} \frac{\varepsilon_{r1}}{1 - \varepsilon_{r1}} = U_i \varepsilon_{r1} )</td>
<td>Mazumder et al. 2009, 114; Kwaau 1992</td>
</tr>
<tr>
<td>Riser top dilute region solids holdup, ( \varepsilon_{sr2} )</td>
<td>( \varepsilon_{sr2} = 2.64 \times 10^{-14} U_{lr}^{-5.343} + 2.57 \times 10^{-5} \varepsilon_{sr1} U_{lr}^{-1.578} )</td>
<td>Mazumder et al. 2009, 114</td>
</tr>
<tr>
<td>Bed expansion index, ( n )</td>
<td>( n = \left( 4.4 + 18 \frac{d_p}{D_c} \right) \text{Re}_t^{-0.01} ) for ( 1 &lt; \text{Re}_t &lt; 200 )</td>
<td>Richardson and Zaki 1954, 35; Kwaau 1992</td>
</tr>
<tr>
<td>Terminal Reynolds, ( \text{Re}_t )</td>
<td>( \text{Re}_t = \frac{U_t d_p \rho}{\mu} )</td>
<td>Fan et al. 1985, 1801</td>
</tr>
</tbody>
</table>
Terminal velocity, \( U_t \)

\[
U_t = \frac{gd_p^2}{18\mu}(\rho_w - \rho)
\]

Fan et al. 1985, 1801

Superficial liquid velocity at \( \varepsilon = 1 \), \( U_i \)

\[
\frac{U_i}{U_t} = 1 - 1.15 \left( \frac{d_p}{D_c} \right)^{0.6}
\]

Khan and Richardson 1989, 111

Separator voidage, \( \varepsilon_c \)

\[
1 - \varepsilon_c = \frac{(D_t / D_c)^2 G_s}{\rho_s U_t}
\]

Zheng and Zhu 2000, 146

Return pipe voidage, \( \varepsilon_{re} \)

\[
1 - \varepsilon_{re} = \frac{(D_t / D_{re})^2 G_s}{\rho_s U_t}
\]

Zheng and Zhu 2000, 146

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Calibrated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant factor defined in Eq. 16b, ( \Psi )</td>
<td>( 3.944 \times 10^{-3} e^{3.9336G_s} )</td>
</tr>
<tr>
<td>Dissociation constant, ( K_d ) (kg/m³)</td>
<td>0.25</td>
</tr>
<tr>
<td>Desorption rate constant of distributor region, ( k_{r1} ) (m/s)</td>
<td>( 5.253 \times 10^{-3} )</td>
</tr>
<tr>
<td>Desorption rate constant of upper dilute region, ( k_{r2} ) (m/s)</td>
<td>( 6 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

Tab. 5: Model parameters used in numerical simulation

<table>
<thead>
<tr>
<th>Operating parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed concentration, ( C_{od} ) (kg/m³)</td>
<td>2</td>
</tr>
<tr>
<td>Solids circulation rate, ( G_s ) (kg/m²/s)</td>
<td>1.24</td>
</tr>
<tr>
<td>Downer superficial liquid velocity, ( U_{ld} ) (m/s)</td>
<td>( 6 \times 10^{-4} )</td>
</tr>
<tr>
<td>Riser superficial liquid velocity, ( U_{lr} ) (m/s)</td>
<td>( 1.13 \times 10^{-2} )</td>
</tr>
<tr>
<td>Amount of dry solid particles, ( S ) (kg)</td>
<td>3</td>
</tr>
</tbody>
</table>

Tab. 6: Base case parameter values